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ENDOCHRONIC MODELING FOR CYCLIC BEHAVIOR OF OVERCONSOLIDATED CLAYS

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ABSTRACT

The constitutive relations based on the endochronic theory have been presented to describe the behavior of overconsolidated clay subjected to cyclic loading. The proposed constitutive law is capable of describing (a) strain softening and hardening, including a softening steady state which is eventually reached as the number of cycles increases; (b) densification and dilatancy; (c) effect of effective confining pressure; (d) effect of overconsolidation ratio OCR. The model is applied to describe the strain-controlled cyclic undrained triaxial test data on Shanghai clay carried out by the second author and strain-controlled undrained simple shear test data on Drammen clay (Andersen et al., 1980). The comparison between the theoretical prediction and the test data shows that the presently proposed model provides a reasonable description for the clay under consideration.

Key words: clay, consolidated undrained shear, constitutive equation of soil, endochronic model, hysteresis, overconsolidation, pore pressure (**IGC**: D6)

INTRODUCTION

In recent years, an understanding of cyclic behaviors of clay has been recognized as one of the most important subjects in geotechnical engineering and many efforts have been made to study this subject. However, because of the complex behavior of clay under cyclic loading, there is relatively little experience and less progress has been made. To date most previous modellings of cyclic loading have been done by use of empirical functions to present shear hysteresis or accumulation of pore pressure. So it is now required to construct a model of cyclic loading which considers some insight into the underlying mechanisms and is not merely based on empirical correlations.

Endochronic theory is considered as an excellent one which can consider the physical mechanisms of material since it takes internal variables into account to describe the microstructural change inside material and takes intrinsic time into account to express the history of strain. This theory has been applied to describe the inelastic behavior of soils under

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cyclic loading by Bažant et al. (1976) and Wu et al. (1983).

Bažant et al.'s cyclic model (1976), with an emphasis directing toward the sand, offers a reasonable description for the soil's response to cyclic loading. However the model uses many parameters and much complex expression of strain hardening softening function. Especially, the constitutive equations are given in a differential form, and the differential form makes it difficult to predict response to large number of cycles.

Wu and Sheu's cyclic model (1983) has an analytical form and uses less parameters and simpler expression of strain hardening softening function. The model, however, is constructed to describe the deviatoric response part of sand only (although a description of the densification part of sand subjected to static loading has been given in the paper written by Wu and Wang (1983)).

In this paper, the new version of endochronic model proposed by authors in the previous paper (1990) is used to investigate the cyclic response of overconsolidated clay. The relation consists of two parts, i.e., the deviatoric response and pore pressure response. An attempt is made to use less parameters, simpler strain hardening softening function, and analytical constitutive equations.

It is noted that there are some imperfections in the original endochronic theory, and some reasonable criticisms were given by Sandler (1978) and Rivilin (1981). However, the relevant improvements have been made by Valanis (1978, 1981), and in this paper, his improved version of endochronic theroy is applied and developed.

EXPERIMENTAL PROGRAM

Strain-controlled cyclic triaxial tests with a sinusoidal strain history were carried out at a frequency of 0.1 Hz on remoulded saturated Shanghai clay, which has a plasticity index of about 12% and a water content of about 27%. Normally consolidated specimens were made by isotropic consolidation for 24 hours under two confining pressures (2 kgf/cm² (196.2 kN/m²) and 4 kgf/cm² (392.4 kN/m²)). Overconsolidated specimens were made by isotropic consolidation under an effective confining pressure of 4 kgf/cm², followed by swelling for 24 hours to give three overconsolidation ratios (1, 2, and 4). Axial strain amplitudes ranged from about 0.5% to 3.7% (see Table 1.).

Table 1. Consolidation and cyclic shear conditions of triaxial shear samples (1 kgf/cm² = 98.1 kN/m²)

OCR	Pre-consolidation	Swelling after pre-consolidation	Strain amplitude	Frequency
	σ_p (kgf/cm²)	σ_c' (kgf/cm²)	ε_0	f (Hz)
1	2	2	0.58%	0.1
			1.22%	0.1
			3.71%	0.1
	4	4	0.53%	0.1
			1.21%	0.1
			3.61%	0.1
2	4	2	1.20%	0.1
4	4	1	1.36%	0.1

These tests were carried out in a servo-controlled triaxial apparatus developed by the Structural Behavior Engineering laboratory (U.S.A.). The above stated cyclic deformation was applied to the specimens, and developed pore pressures, axial loadings and strains were recorded for 100 cycles.

NORMALIZATION IN THE CYCLIC LOADING TEST

In static test, a normalized soil behavior has been found by Ladd and Foott (1974). That is, the response curve, normalized with respect to initial effective consolidation stress σ_c', is almost the same for the different values of σ_c'. Such a normalization can also be seen in the cyclic loading test. Fig. 1 shows the relations between the number of cycles N and the normalized peak deviatoric stress $\hat{s}/\sigma_c'$. The specimens were normally consolidated under effective confining pressures of 2 kgf/cm² and 4 kgf/cm² and then subjected to cyclic loadings

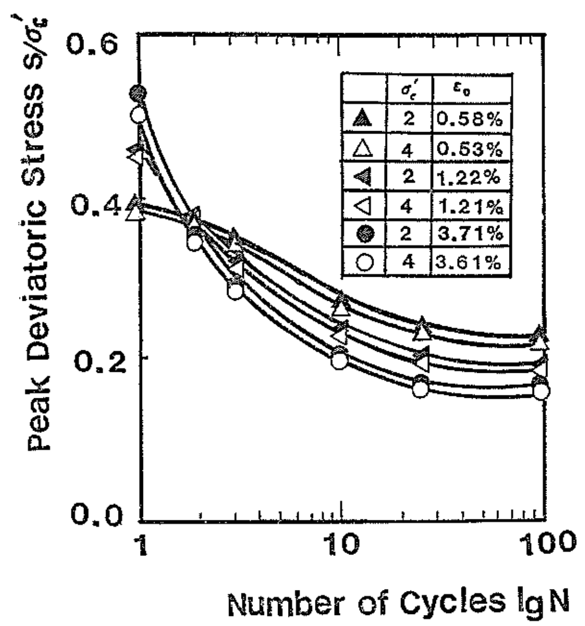

Fig. 1. Effect of effective confining pressure on deviatoric stress

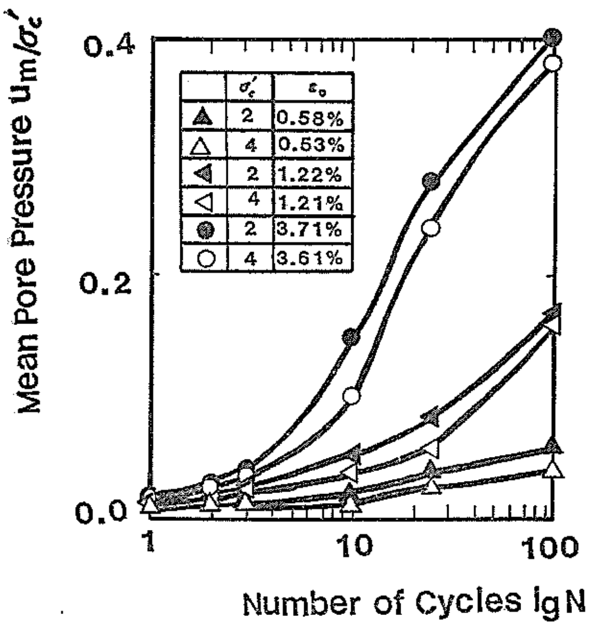

Fig. 2. Effective of effective confining pressure on pore pressure

with a frequency of 0.1 Hz and cyclic constant axial strains of about 0.5%, 1.2%, and 3.6%. From Fig. 1 it can be seen that for a given axial strain amplitude ε_0, curves with different σ_c' can be combined into a curve. This property can also be noted in pore pressure response (see Fig. 2). Therefore in the following sections, the deviatoric stress $\underline{s}$ and effective volumetric stress σ_{kk}' can be normalized by terms $\underline{s}/\sigma_c'$ and σ_{kk}'/σ_c', and the corresponding constitutive equations can be written in a normalized form, which makes the parameters appearing in the constitutive equations independent of σ_c' and thus facilitates the determination of parameters.

The endochronic constitutive relations for overconsolidated clay consist of the deviatoric and the volumetric parts, and have been derived by authors (Imai and Xie, 1990) as follows:

$$\bar{\underline{s}} = 2G\rho_0 \frac{d\underline{e}^P}{dZ_s} + \int_0^{Z_s} \rho_1(Z_s - Z_s') \frac{d\underline{e}^P(Z_s')}{dZ_s'} dZ_s' + \mu_2 \underline{e}^P \qquad (1)$$

$$\bar{\sigma}_{kk}' = A\varepsilon - \sum_d k^d \{\varepsilon - \varepsilon(0)\exp(-\lambda^d Z_D) - \int_0^{Z_D} \exp[-\lambda^d (Z_D - Z_D')] \cdot \frac{d\varepsilon(Z_D')}{dZ_D'} dZ_D'\} + \sum_d \gamma^d [1 - \exp(-\rho^d Z_D)] \qquad (2)$$

In which $\bar{\underline{s}} = \underline{s}/\sigma_c'$; $\bar{\sigma}_{kk}' = \sigma_{kk}'/\sigma_c'$; $\rho_1(Z) = \sum_{r=1}^{s-1} R_r \exp(-\beta_r Z)$; $\underline{e}^P = \underline{e}^D - \bar{\underline{s}}/(2G)$; $\varepsilon = \varepsilon_{kk}/3$; ρ_0, R_r, β_r, μ_2, A, k^d, λ^d, γ^d, and ρ^d are positive material constants; $2G$ is shear modulus; $\underline{e}^D$ is total deviatoric strain tensor; and Z_s and Z_D are, respectively, intrinsic times associated with the deviatoric and volumetric response.

The proposed constitutive Eqs. (1) and (2) were derived, in general, for overconsolidated clays. So it may be used in either effective stress analysis or total stress analysis. In the case of effective stress analysis the data of drained test are used to evaluate the required parameters, and in the case of total stress analysis undrained tests are used to determine the parameter values. For response of clays subjected to cyclic load, loss of pore water from the soil layer might be little because long-time period is required for perfect drainage except for very slow frequency. Therefore only situation involving undrained conditions will be considerd herein.

CYCLIC DEVIATORIC RESPONSE

Strain Hardening-softening Function

When the strain hardening-softening property of a material is considered, the intrinsic time Z_s in Eq. (1) could be expressed as

$$d\zeta_s = f(Z_s)dZ_s \quad (3)$$

where $f(Z_s)$ is the strain hardening-softening function; and ζ_s is governed by

$$d\zeta_s^2 = d\underline{\varepsilon}^P \cdot d\underline{\varepsilon}^P \quad (4)$$

In the previous paper (Ref. 4), the $f(Z_s)$ was assumed as

$$f(Z_s) = (1+\alpha_s Z_s)\exp(-\beta_s Z_s) \quad (5)$$

and it has been shown that the expression is capable of not only describing the first hardening followed by softening property of overconsolidated clay subjected to static loading, but also making that the corresponding deviatoric constitutive equation is expressed in a closed form. However, the further study found that the $f(Z_s)$ with the form of Eq. (5) is not suitable for more complex loadings such as cyclic loading. Because in such a case as time goes on, the overconsolidated clay experiences a process of first strain hardening then softening and lastly tending steady, and the steady state can not be described by Eq. (5).

Here, a new strain hardening-softening function is proposed for cyclic loading. First the $f(Z_s)$ is assumed as a product of hardening function $f_h(Z_s)$ and softening one $f_s(Z_s)$ by

$$f(Z_s) = f_h(Z_s) \cdot f_s(Z_s)$$

Considering that a hardening-softening function to be chosen should make Eq. (1) expressed in a closed forms, the several possible choices for $f_h(Z_s)$ and $f_s(Z_s)$ are $a \pm b\ \exp(-\beta Z_s)$, $a \pm b \cdot Z_s$, and $a \cdot \exp(\pm \beta Z_s)$. The second and the third forms are not suitable because they will give a conclusion that the peak stress of each cycle decreases or increases monotonically with time. So the $f(Z_s)$ is proposed as follows:

$$f(Z_s) = [a - b \cdot \exp(-\beta_1 Z_s)][c + d \cdot \exp(-\beta_2 Z_s)] \quad (6)$$

in which, a, b, c, and $d>0$. Since $f(0)=1$, we obtain:

$$b = a - 1/(c+d)$$

By substituting the relation into Eq. (6), the $f(Z_s)$ can be rewritten as

$$f(Z_s) = \{a - [a - 1/(c+d)]\exp(-\beta_1 Z_s)\}[c + d\exp(-\beta_2 Z_s)] \quad (7)$$

where, $a > 1/(c+d)$, which is corresponding to $b>0$.

From Eq. (7) it can be seen that $f(Z_s) = a \cdot c$ for $Z_s \to \infty$, and that if β_1 is chosen enough larger than β_2, the function $f(Z_s)$ will describe the initially hardening then sofening until a steady state is reached.

Constitutive Equation for Cyclic Deviatoric Response

For the case of one dimension, if only one term of the function $\rho_1(Z)$ is employed, Eq. (1) can be written as

$$\tilde{s} = 2G\rho_0 \frac{d\varepsilon^P}{dZ_s} + \mu_1 \int_0^{Z_s} \exp[-\alpha(Z_s - Z_s')] \frac{d\varepsilon^P(Z_s')}{dZ_s'} dZ_s' + \mu_2 \varepsilon^P \quad (8)$$

where μ_1 and α are positive constants, and Eq. (4) can be given by

$$d\zeta_s^2 = d\varepsilon^P \cdot d\varepsilon^P \quad (9)$$

Eqs. (3), (7), (8), and (9) are now combined to describe the hysteretic curves of overconsolidated clay subjected to cyclic shear strain.

When $Z_s^{i*} < Z_s < Z_s^{(i+1)*}$, it is obtained from Eq. (9) and Fig. 3 that

$$d\varepsilon^P / d\zeta_s = d\varepsilon^P / |d\varepsilon^P| = (-1)^i \quad (10)$$

Eq. (10) may be integrated to yield the following relation between ε^P and ζ_s

$$\begin{aligned}\varepsilon^P &= \int_0^{\varepsilon^P} d\varepsilon^P = \sum_{j=0}^{i-1} \int_{\varepsilon^{Pj*}}^{\varepsilon^{P(j+1)*}} d\varepsilon^P + \int_{\varepsilon^{Pi*}}^{\varepsilon^P} d\varepsilon^P \\ &= \sum_{j=0}^{i-1} \int_{\zeta_s^{j*}}^{\zeta_s^{(j+1)*}} (-1)^j d\zeta_s + \int_{\zeta_s^{i*}}^{\zeta_s} (-1)^i d\zeta_s \\ &= \sum_{j=0}^{i-1} [(-1)^j (\zeta_s^{(j+1)*} - \zeta_s^{j*})] + (-1)^i (\zeta_s - \zeta_s^{i*})\end{aligned}$$

that is

$$\varepsilon^P = (-1)^i \zeta_s + 2\zeta_s^* - \cdots + 2\zeta_s^{i*}(-1)^{i+1}, \quad Z_s^{i*} < Z_s < Z_s^{(i+1)*} \quad (11)$$

Similarly, the integral part of Eq. (8) can be integrated by using Eq. (10) as follows:

$$\mu_1\int_0^{Z_s}\exp[-\alpha(Z_s-Z_s')]\frac{d\varepsilon^P(Z_s')}{dZ_s'}dZ_s'$$
$$=\mu_1\int_0^{Z_s}\exp[-\alpha(Z_s-Z_s')]\frac{d\varepsilon^P(Z_s')}{d\zeta_s'}\cdot\frac{d\zeta_s'}{dZ_s'}dZ_s'$$
$$=\mu_1\exp(-\alpha Z_s)\Big[\sum_{j=0}^{i-1}\int_{Z_s^{j*}}^{Z_s^{(j+1)*}}\exp(\alpha Z_s')\cdot(-1)^j f(Z_s')dZ_s' + \int_{Z_s^{i*}}^{Z_s}\exp(\alpha Z_s')\ (-1)^i f(Z_s')dZ_s'\Big]$$
$$=\mu_1\exp(-\alpha Z_s)[-f_a(0)+2f_a(Z_s^*)-2f_a(Z_s^{2*})+\cdots+2(-1)^{i+1}f_a(Z_s^{i*})+(-1)^i f_a(Z_s)];$$
$$Z_s^{i*}<Z_s<Z_s^{(i+1)*} \qquad (12)$$

in which $f_a(Z)$ is equal to $\int\exp(\alpha Z)f(Z)dZ$, and is integrable. Correspondently, Eq. (8) can be written as:

$$\bar{s}=2G\rho_0\frac{d\varepsilon^P}{d\zeta_s}\cdot\frac{d\zeta_s}{dZ_s}+\mu_2\varepsilon^P+\mu_1\int_0^{Z_s}\exp[-\alpha(Z_s-Z_s')]\frac{d\varepsilon^P(Z_s')}{dZ_s'}dZ_s' \qquad (13)$$

which, when combined with Eq. (10) and (12), gives:

$$\bar{s}=2G\rho_0(-1)^i f(Z_s)+\mu_2\varepsilon^P+\mu_1\exp(-\alpha Z_s)\cdot[-f_a(0)+2f_a(Z_s^*)+ +\cdots+2(-1)^{i+1}f_a(Z_s^{i*})+(-1)^i f_a(Z_s)];\ Z_s^{i*}<Z_s<Z_s^{(i+1)*} \qquad (14)$$

The constitutive equation (14) is given in a closed form and has two advantages. The first one is that the parameters can easily be valued only by solving a group of equations and unnecessarily by step-by-step integration, and the second one is that the computing time does not increase obviously with the number of cycle, therefore it can be used to predict the response for arbitrarily large number of cycles without difficulty.

It can also be shown that Eq. (14) includes constitutive laws during the initial stage of elastic loading and the later stages of elastic

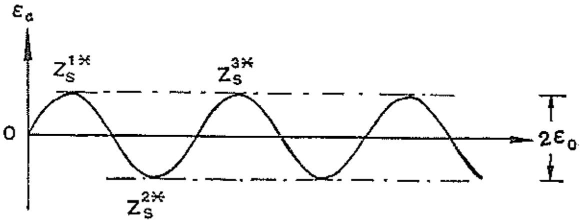

Fig. 3. Time history of cyclic loading

unloading and reloading. Z_s^{i*} in Fig. 3 is the point where the direction of strain is reversed and elastic deformation takes place. Assuming $Z_{s+}^{i*}=\lim_{\delta\to 0}(Z_s^{i*}+\delta)$ and $Z_{s-}^{i*}=\lim(Z_s^{i*}-\delta)$, thus $Z_s^{i*}<Z_{s+}^{i*}<Z_s^{(i+)*}$ and $Z_s^{(i-1)*}<Z_{s-}^{i*}<Z_s^{i*}$. It is obvious in Eq. (14) that only the first term is discontinuous at the point $Z_s=Z_s^{i*}$, and can be expressed by:

$$2G\rho_0(-1)^i f(Z_s) = \begin{cases} 2G\rho_0(-1)^i f(Z_s^{i*})\ ; & Z_s=Z_{s+}^{i*} \\ 2G\rho_0(-1)^{i-1} f(Z_s^{i*})\ ; & Z_s=Z_{s-}^{i*} \text{ and } i>0 \\ \ ; & Z_s=Z_{s-}^{i*} \text{ and } i=0 \end{cases}$$

therefore

$$\bar{s}(Z_{s+}^{i*})-\bar{s}(Z_{s-}^{i*}) = \begin{cases} (-1)^i 4G\rho_0 f(Z_s^{i*})\ ; & i>0 \\ 2G\rho_0\ ; & i=0 \end{cases} \qquad (15)$$

It describes that in elastic stages an elastic jump of stress exists in $\bar{s}-\varepsilon^P$ curve and the value of jump varies with hardening-softening function $f(Z_s)$. This is in accordance with the observed behavior of clay. The whole process of the response of clay subjected to cyclic loading may, therefore, be simulated by Eq. (14).

Determination of Material Parameters

Triaxial undrained cyclic test TUC is used to determine the material constants. In the case of the TUC test

$$\bar{S}=\frac{2}{3}(\bar{\sigma}_a-\bar{\sigma}_r),\quad \varepsilon^D=\varepsilon_a-\frac{\varepsilon_{kk}}{3}=\varepsilon_a-\frac{\varepsilon_{kk}{}^0}{3}=\bar{\varepsilon}_a \qquad (16)$$

Where $\bar{\sigma}_a$ and $\bar{\sigma}_r$ are normalized axial and confining stress, ε_a is axial strain including consolidation and $\bar{\varepsilon}_a$ is that after the start of shearing, ε_{kk} is volumetric strain including consolidation and $\varepsilon_{kk}{}^0$ is that before the start of shearing.

By using Eq. (16), Eqs. (9) and (14) can be written as:

$$d\zeta_s{}^2 = d\bar{\varepsilon}_a{}^P\, d\bar{\varepsilon}_a{}^P \qquad (17)$$

and
$$\bar{s} = 2G\rho_0(-1)^i f(Z_s) + \mu_2\bar{\varepsilon}_a{}^P + \mu_1 \exp(-\alpha Z_s)[-f_a(0) + 2f_a(Z_s{}^*) + \cdots + 2(-1)^{i+1} f_a(Z_s{}^{i*}) + (-1)^i f_a(Z_s)]\ ;\ Z_s{}^{i*} < Z_s < Z_s{}^{(i+1)*} \qquad (18)$$

The material parameters can be determined in the manner similar to the previous paper (Imai and Xie, 1990); that is, the 2G, μ_1, μ_2, and α are connected indirectly with OCR by the following Eq. (19) derived from Eq. (18):

$$\begin{aligned} &2G\rho_0 f'(0) + \mu_1 + \mu_2 = 2G^P \\ &\{2G\rho_0 f(Z_s) + \mu_2\bar{\varepsilon}_a{}^P + \mu_1 \exp(-\alpha Z_s)\cdot [f_a(Z_s) - f_a(0)]\}_{\bar{\varepsilon}_a{}^P = \bar{\varepsilon}_0{}^P} = \bar{S}_u \qquad (19)\\ &\{2G\rho_0 f'(Z_s) + (\mu_2+\mu_1) f(Z_s) - \alpha\mu_1 \exp(-\alpha Z_s)\cdot [f_a(Z_s) - f_a(0)]\}_{\bar{\varepsilon}_a{}^P = \bar{\varepsilon}_0{}^P} = 2G_P{}^P \end{aligned}$$

where $\bar{\varepsilon}_0{}^P$ = value of plastic deviatoric strain corresponding to total deviatoric strain amplitude $\bar{\varepsilon}_0{}^D$ and $f'(Z_s) = \dfrac{df(Z_s)}{dZ_s}$. $2G^P$, $2G_P{}^P$, and $\bar{s}_u$ are, respectively, initial slope, final slope, and peak shear stress of the normalized stress–plastic strain curve in first 1/2 cycle. For a given strain amplitude, the values of $2G^P$, $2G_P{}^P$, and $\bar{S}_u$ could be assumed to depend on OCR. This is logical since the rigidity and strength of the solid skeleton depend on the numbers and areas of interparticle contacts, which in turn depend on, in general, OCR, σ_c', and $\bar{\varepsilon}_0$, the latter two, however, can be excluded, considering that the $2G^P$, $2G_P{}^P$ and $\bar{S}_u$ are defined about normalized curve and $\bar{\varepsilon}_0$ is a constant. Thus we may introduce:

$$\begin{aligned} 2G^P &= f_1{}^D\ (\mathrm{OCR}) \\ 2G_P{}^P &= f_2{}^D\ (\mathrm{OCR}) \qquad (20)\\ \bar{S}_u &= f_3{}^D(\mathrm{OCR}) = (\bar{S}_u)_{NC}\cdot \mathrm{OCR}^m \end{aligned}$$

where functions $f_1{}^D$, $f_2{}^D$ and m are determined by experimental data, and as an approximation, it is here assumed that $2G$ is expressed by the same function as $\bar{S}_u$, that is

$$2G = (2G)_{NC}\cdot \mathrm{OCR}^m \qquad (21)$$

The remaining 5 parameters a, c, d, β_1, and β_2 are related to isotropic strain hardening softening and are considered to be the same for various values of OCR. Therefore, when the material functions $f_1{}^D$, $f_2{}^D$, and m in Eq. (21) have been known, all of the material parameters in Eq. (18) can be determined only by the experimental curve of OCR=1, and further in the first $1\frac{1}{2}$ cycle, as follows:

1) When OCR=1 and $N<1\frac{1}{2}$, the value of $(2G)_{NC}$ can be obtained by measuring the slope of unloading and reloading of the $\bar{s}\sim\bar{\varepsilon}_a$ curve, and ρ_0 is determined from initial yield stress $\sigma_y = (2G)_{NC}\rho_0$ of the same curves; $(\bar{S}_u)_{NC}$ is evaluated from the $\bar{s}\sim\bar{\varepsilon}_a{}^P$ curve; the value of involved constants $2G^P$ and $2G_P{}^P$ are obtained by Eq. (20). Then for a group of assumed values of a, c, d, β_1, and β_2, the constants of μ_1, μ_2, and α can be obtained from Eq. (19). Next, for a value of Z_s, $\bar{s}$ can be calculated from Eq. (18) and $\bar{\varepsilon}_a{}^P$ can be calculated from Eqs. (3), (6), and (11), thus the theoretic curve of $\bar{s}\sim\bar{\varepsilon}_a{}^P$ is obtained. Then the values of a, c, d, β_1, and β_2 are modified so that the theoretical curve fits the test data well.

2) When OCR=1 and $N>1\frac{1}{2}$, the response is predicted by Eq. (18) without changing the above determined parameters.

3) When OCR$\neq$1, keeping a, c, d, β_1, and β_2 unchanged and by using Eqs. (19), (20), and (21) the material parameters, μ_1, μ_2 and α are obtained.

Fig. 4 gives the comparisons between computed hysteresis loops and actual test data (solid curves indicated predictions from the model). Although the model can be used to determine predictions for arbitrarily large number of cycles without difficulty, the test data show that the variation in hysteresis loop is small for cycles of larger than 100, so Fig. 4 gives only comparisons for cycles of less than 100.

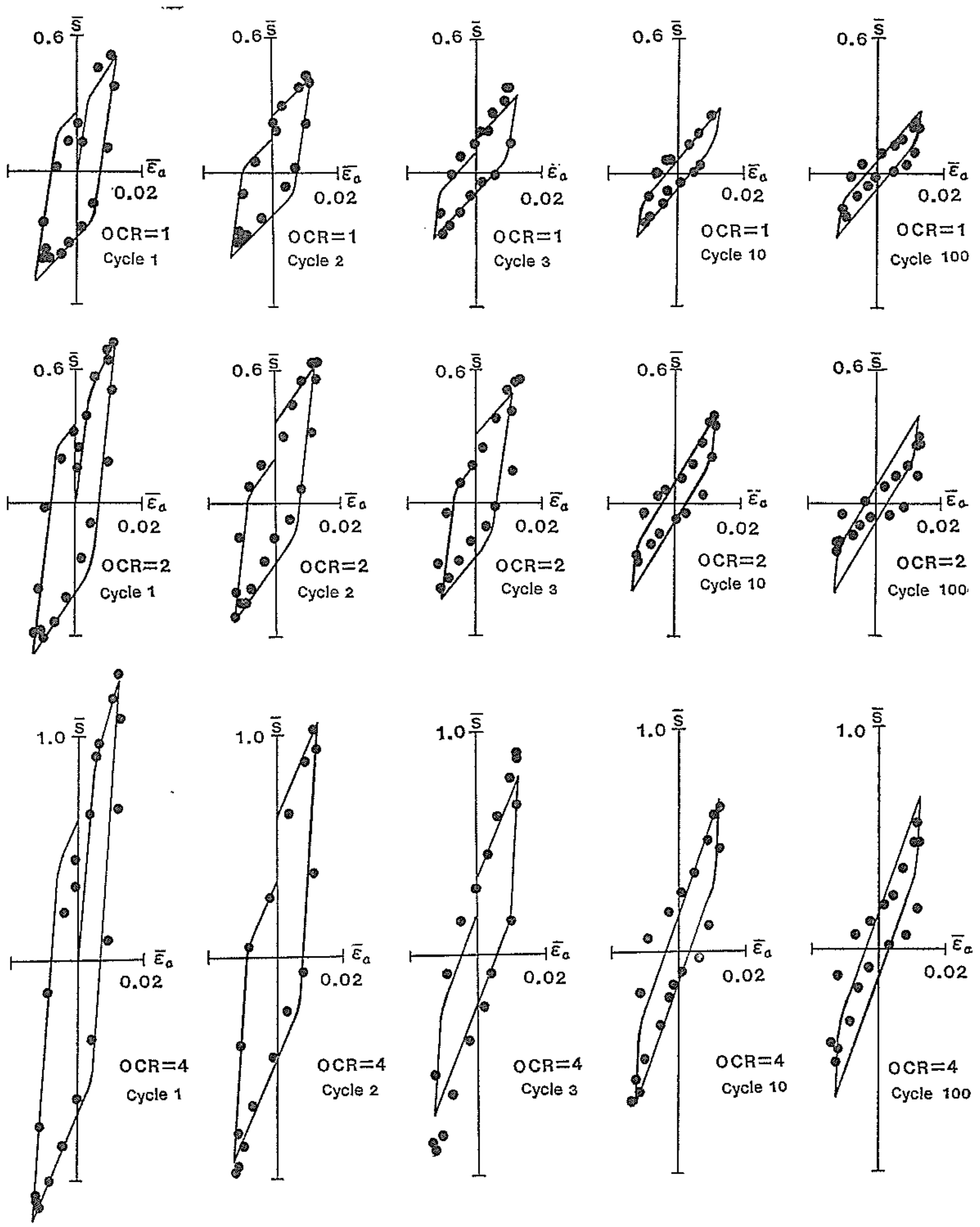

Fig. 4. Hysteresis loop data for Shanghai clay with theoretical response

The material function $f_1{}^D$, $f_2{}^D$, and m, in this case, are found to be

$$f_1{}^D = 40(1+\text{OCR})$$
$$f_2{}^D = 7.4 + 1.875 \cdot \text{OCR}$$
$$m = 0.0845 \cdot \text{OCR} + 0.3006$$

and, $(\bar{S}_u)_{NC} = 0.52$, $(2G)_{NC} = 92.3.$, in which NC denotes normal consolidation.

The constant parameters listed in Table-2 are the same for all the OCRs, while the material parameters are given in Table-3.

From Fig. 4, it is seen that the endochronic

Table 2. Constant parameters for examples illustrated in Fig. 4

a	c	d	β_1	β_2	ρ_0
3	0.08	0.42	200	20	0.00325

Table 3. Material parameters for examples illustrated in Fig. 4

OCR	μ_1	μ_2	α
1	37.7	22.3	170524.0
2	54.5	31.0	178098.6
4	87.0	52.5	181634.5

dynamic model for overconsolidated clay provides a reasonably good representation of the observed experimental behavior.

CYCLIC ACCUMULATION OF PORE PRESSURE

In undrained cyclic loading tests, as volumetric strains are prevented, a gradual accumulation of pore pressure is observed. It has been shown (Sangrey, 1968) that under certain conditions the increase of pore pressure may reduce effective stresses sufficiently for failure to occur. So it is important to simulate the development of pore pressure in cyclic loading.

Mean Pore Pressure Response

The volumetric deformation consists of, in general, two components. The first is due to isotropic consolidation, and the second is due to shear. Because overconsolidated clays are considered here, the volumetric strain caused by volumetric stress could be assumed completely reversible and only the inelastic volumetric deformation caused by shear is considered. For the case of undrained triaxial test, the pore pressure rersponse can be derived from Eq. (2) by using relation $\bar{\sigma}_{kk}'=\bar{\sigma}_{kk}-3\bar{u}=3+\frac{3}{2}\bar{s}-3\bar{u}$ as follows (Imai and Xie, 1990):

$$\bar{u}=\bar{s}/2-R[1-\exp(-\rho Z_D)]+C[1-\exp(-\lambda Z_D)] \tag{22}$$

where, Z_D is defined by

$$dZ_D=d\zeta_D/(1+\xi\zeta_D) \quad ; \ (\xi>0) \tag{23}$$

In Wu and Wang's static load model (1983), the intrinsic time corresponding to densification is defined as $d\zeta_D=K_D(\sigma_c)\,|d\varepsilon^D|$ based on the observed test results that the densification grows even when unloading takes place in shear. Considering that the constitutive equations presented is written in a normalized form about σ_c', a slightly different expression for $d\zeta_D$ is taken as:

$$d\zeta_D{}^2=d\varepsilon^D\cdot d\varepsilon^D \tag{24}$$

For the special case of undrained triaxial tests, $d\zeta_D$ could be expressed as follows by using Eq. (16):

$$d\zeta_D{}^2=d\bar{\varepsilon}_a\cdot d\bar{\varepsilon}_a \tag{25}$$

Then by substituting Eq. (23) into Eq. (22), the following equation is given

$$\bar{u}=\bar{s}/2-R[1-(1+\xi\zeta_D)^{-\rho}]+C[1-(1+\xi\zeta_D)^{-\lambda}] \tag{26}$$

In strain-controlled dynamic tests on clay, for example, those described by Taylor and Bacchus (1969), it can be seen that the pore-pressure cyclic fluctuations and axial load cyclic fluctuations were similar in form, in phase with each other, and that the magnitude of the pore pressure fluctuation was close to one third of the magnitude of the deviator stress fluctuation. Therefore the only the mean pore-pressure response, rather than pore-pressure itself, is of interest and is studied in this section.

When sinusoidal strain-controlled cyclic loads are applied, $\bar{\varepsilon}_a=\bar{\varepsilon}_0\sin\omega t$, and the value of ζ_D at the end of the Nth cycle is

$$\zeta_D=4\bar{\varepsilon}_0N$$

Accordingly, after N cycles of shear, Eq. (26) can be expressed as

$$\bar{u}_m=\bar{s}_m/2-R[1-(1+4\bar{\varepsilon}_0\xi N)^{-\rho}]+C[1-(1+4\bar{\varepsilon}_0\xi N)^{-\lambda}] \tag{27}$$

in which $\bar{u}_m$ and $\bar{s}_m$ are, respectively, the mean normalized pore pressure and the mean normalized shear stress.

Generally, in triaxial samples the behavior is different in compression and extension and a mean shear stress $\bar{s}_m$ different from zero will develop even if the applied strain history is symmetrical.

But the $\bar{s}_m$ is always rather small compared with $\bar{u}_m$, and as a simplification, $\bar{s}_m=0$ is assumed herein. Thus Eq. (27) can be written as

$$\bar{u}_m=-R[1-(1+4\bar{\varepsilon}_0\xi N)^{-\rho}] +C[1-(1+4\bar{\varepsilon}_0\xi N)^{-\lambda}] \qquad (28)$$

Test results show that the mean pore pressure varies up to a large number of N, and the results of the 100 applied strain cycles test conducted in this study are not enough to completely express pore pressure behavior, so as a replacement the data conducted by Andersen et al. (1980) are used to demonstrate the capability of the proposed model. These tests (1980) were performed in a simple shear apparatus. Undisturbed Plastic Drammen clay was used. The tests were carried out under strain-controlled and undrained conditions with a sinusoildal pulse form at a frequency of 0.1 Hz. The shear stress and pore pressure to deform the sample were then recorded.

For the case of simple shear undrained test, Eq. (25) can be written as

$$d\zeta_D=|d\gamma| \qquad (29)$$

where, γ is the total shear strain. Due to $\varepsilon=\varepsilon(0)=\text{Const.}$, and specifying that $d=1$ for simplicity, Eq. (2) becomes:

$$\bar{\sigma}_{kk}'=A\varepsilon(0)-k[\varepsilon(0)-\varepsilon(0)\exp(-\lambda Z_D)] +\gamma[1-\exp(-\rho Z_D)] \qquad (30)$$

Since $Z_D=0$, and $\bar{\sigma}_{kk}'=(2\sigma_r'+\sigma_a')/\sigma_v'=(2k_0\sigma_v'+\sigma_v')/\sigma_v'$, Eq. (30) yields:

$$\varepsilon(0)=(2k_0+1)/A \qquad (31)$$

in which, σ_r' and σ_a' are, respectively, the effective radius and axial stress, σ_v' is the initial effective vertical stress, and k_0 is the coefficient of earth pressure at rest. Combining Eqs. (30) with (31), the following equation is obtained

$$\bar{\sigma}'=(2k_0+1)/3+R_1[1-\exp(-\rho Z_D)] -C_1[1-\exp(-\lambda Z_D)] \qquad (32)$$

where, $C_1=k(2k_0+1)/(3A)$, $R_1=\gamma/3$. Then by appliying the effective stress principle and substituting Eq. (23) into Eq. (32), the pore pressure can be expressed as

$$\bar{u}=u/\sigma_v'=\bar{\sigma}-(2k_0+1)/3-R_1[1-(1 +\xi\zeta_D)^{-\rho}]+C_1[1-(1+\xi\zeta_D)^{-\lambda}] \qquad (33)$$

If a simplifying assumption that the stresses, σ_a and σ_r, remain a constant k_0 relationship during simple shear is introduced, $\bar{\sigma}=\sigma/\sigma_v'=\sigma_a(2k_0+1)/(3\sigma_v')=(2k_0+1)/3$ (where, σ_a is set equal to σ_v' due to the constant vertical stress condition in the case of simple shear), and Eq. (33) can be rewritten by:

$$\bar{u}=-R_1[1-(1+\xi\zeta_D)^{-\rho}]+C_1[1-(1+\xi\zeta_D)^{-\lambda}] \qquad (34)$$

When sinusoidal strain-controlled cyclic loads are applied, $\gamma=\gamma_0\sin\omega t$ (γ_0 is the amplitude of total shear strain), Eq. (34) after N cycles of shear can be expressed as:

$$\bar{u}_m=-R_1[1-(1+4\gamma_0\xi N)^{-\rho}] +C_1[1-(1+4\gamma_0\xi N)^{-\lambda}] \qquad (35)$$

Determination of Material Parameters

When the mean pore pressure of each cycle increases monotonically with the increment of N, by taking ρ enough large to yield $(1+4\gamma_0\xi N)^{-\rho}=0$ for $N>0$, Eq. (35) can be simplified as follows;

$$\bar{u}_m=-R_1+C_1[1-(1+4\gamma_0\xi N)^{-\lambda}]\ ;\ N>0 \qquad (36)$$

The involved constants are $2G_{m1}(=d\bar{u}_m/(dN)|_{N=1})$, $\bar{u}_{m1}(\bar{u}=_m|_{N=1})$, and $\bar{u}_{mp}(=\bar{u}_m|_{N=NP}$, N_P is the point where $\bar{u}_m$ tends to a steady state and is taken as 2000 for the present data). They can be found, from the test data, to be connected with OCR by the following relations:

$$\begin{aligned} 2G_{m1}&=0.0185\cdot\text{OCR}^2-0.0755\cdot\text{OCR}+0.116\\ \bar{u}_{m1}&=-\frac{1}{12}\text{OCR}+\frac{1}{3} \qquad (37)\\ \bar{u}_{mp}&=-0.42\cdot\lg\text{OCR}+0.62 \end{aligned}$$

While R_1, C_1, and λ are connected with $2G_{m1}$, $\bar{u}_{m1}$, and $\bar{u}_{mp}$ by the following Eq. (38) derived from Eq. (36)

$$\begin{aligned} -R_1+C_1[1-(1+4\gamma_0\xi)^{-\lambda}]&=\bar{u}_{m1}\\ -R_1+C_1[1-(1+8000\gamma_0\xi)^{-\lambda-1}]&=\bar{u}_{mp} \qquad (38)\\ 4\gamma_0\xi\lambda C_1(1+4\gamma_0\xi)^{-\lambda-1}&=2G_{m1} \end{aligned}$$

The constant parameter ξ is obtained by try and error until the time good fit with the data of OCR=1 is reached, and is kept unchanged for different OCRs.

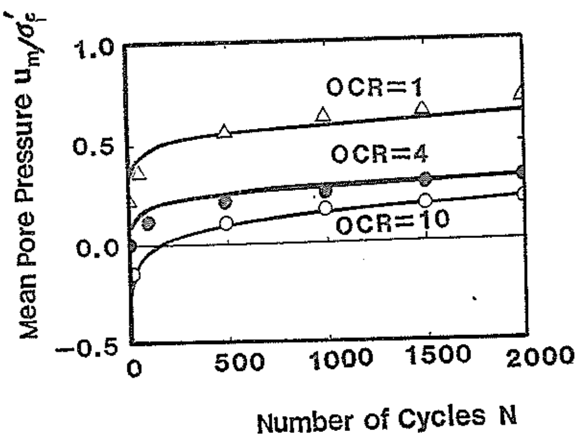

Fig. 5. Pore pressure data for Drammen clay (after Andersen et al., 1980) and predicted responses

Table 4. Material parameters for examples illustrated in Fig. 5

OCR	R_1	C_1	λ
1	0.00149	2.093	0.03325
4	0.1276	5.1438	0.00825
10	0.8816	2.3484	0.05825

Fig. 5 gives the comparisons between the predicted curves by Eq. (36) and the test data. The constant parameter ξ is taken as 1000, and the material parameter is shown in Table 4.

CONCLUSIONS

When constituting a model based on endochronic theory, the following three points are important. The first is to select suitable internal variables to reflect the basic mechanism of general material, the second is to constitute an appropriate strain hardening–softening function to describe the response behavior of the material, and the final is to find a way to determine the values of parameters. In this paper, followed by the previous train of thought presented by authors (1990), the endochronic constitutive relations for overconsolidated clay subjected to cyclic loading are derived by using a new strain hardening–softening function.

First, the reasonableness of normalization in the cyclic loading test is demonstrated by test data, thus the constitutive equations can be expressed in a normalization form, thereby avoiding effective confining pressure to appear in model as a variable.

Next, a suitable strain hardening–softening function is proposed. It not only describes the characteristics of initial hardening followed by softening finally converged to some steady state for a clay subjected to cyclic constant strain, but also gives the corresponding constitutive equation which is expressed in a closed form.

Then, by use of the relations between some constants, which describe main characters of hysteresis curve in first 1/2 cycle such as $\bar{S}_u$, $2G^P$, $2G_P{}^P$ etc., and OCR, the material parameters μ_1, μ_2, and α can be expressed indirectly as the functions of OCR. In this way, if the relations have been found, only based on the experimental data for the first one and half cycles of normally consolidated clays, the model can give predictions for the cases of OCR=1 and $N>1\frac{1}{2}$ and the cases of OCR$\fallingdotseq$1. The material parameters in pore pressure response are obtained in a similar manner.

To demonstrate the capability and reasonableness of the model, strain-controlled cyclic triaxial tests on overconsolidated samples for Shanghai clay were conducted, and a set of data from the reference was also taken. It is observed that good agreement between the predicted value and test data nas been achieved for mean pore pressure response, and that the predicted curve for hysteresis response reasonably agrees with the test data on the whole. But in the case of cycle 100, the predicted curve could not express "S" shape of the test data, mainly because the strain amplitude is too small so that the variation of stain hardening-softening function is not evident.

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NOTATIONS

$\underset{\sim}{\varepsilon}^D, \underset{\sim}{\varepsilon}^P$ = total and plastic deviatoric strain

$\varepsilon^D, \varepsilon^P, \varepsilon_0^D, \varepsilon_0^P$ = total and plastic deviatoric strain for one dimensional case and corresponding amplitude

$\varepsilon_{kk}, \varepsilon$ = total volumetric and total mean strain

ε_{kk}^0 = total volumetric strain before start of shearing

ε_a = total axial strain

$\bar{\varepsilon}_a, \bar{\varepsilon}_0$ = total axial strain after start of shearing and its amplitude

γ, γ_0 = total shear strain and its amplitude

$\underset{\sim}{s}, \bar{\underset{\sim}{s}}$ = deviatoric stress ($\bar{\underset{\sim}{s}} = \underset{\sim}{s}/\sigma_c'$)

$\bar{s}, \hat{s}$ = deviatoric stress and peak one for one dimensional case

σ_r', σ_a' = radius and axial effective stress

$\sigma_{kk}, \bar{\sigma}_{kk}', \sigma, \bar{\sigma}'$ = volumetric and mean stress

σ_y = yield stress

σ_v', σ_c' = vertical and confined initial effective stress

$\bar{s}_m, \bar{u}_m$ = mean shear stress and mean pore pressure

$A, \rho^d, \gamma^d, k^d, \lambda^d$ = constants of volumetric equation

$Z_s, Z_D, \zeta_s, \zeta_D$ = intrinsic times

Z_s^{N*}, ζ_s^{N*} = Value of instrinsic times at Nth strain reversal

$f(Z_s)$,

$f_h(Z_s), f_s(Z_s)$ = hardening-softening function, hardening part, softening part

α_s, β_s = parameters of hardening-softening function associated with static loading

a, c, d = parameters of hardening-softening function associated with cyclic loading

ξ = parameter related to densification

$\rho_1(Z), R_r, \beta_r$ = Kernel function of deviatoric equation and its parameters

$\alpha, \rho_0, \mu_1, \mu_2$ = parameters associated with deviatoric response

R, C, ρ, λ = parameters associated with pore pressure response for triaxial test

R_1, C_1, ρ, λ = parameters associated with pore pressure response for simple shear test

$f_a(Z) = \int \exp(\alpha Z) f(Z)\, dZ$

m = material constant

f_1^L, f_2^D, f_3^D = material functions

$\bar{S}_u = (\bar{s})_{\bar{\varepsilon}_a^P = \bar{\varepsilon}_0^P}$

$2G^P = (ds/d\bar{\varepsilon}_a^P)_{\bar{\varepsilon}_a^P = 0}$

$2G_P^P = (ds/d\bar{\varepsilon}_a^P)_{\bar{\varepsilon}_a^P = \bar{\varepsilon}_0^P}$

$2G_{m1} = (d\bar{u}_m/dN)_{N=1}$

$\bar{u}_{m1} = \bar{u}_m|_{N=1}$

$\bar{u}_{mp} = \bar{u}_m|_{N=N_P}$, N_P is the point where $\bar{u}_m$ tends to steady state

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